2nd Law of Thermodynamics Solutions

1a) HHHH HHHH HHHHH
    HHT THHH HHTT
    HTH THTT HHTT
    HTH THTT HHTT
    HTH THTT HHTT
    HTH THTT HHTT
    HTH THTT HHTT

b) four heads, one tail
   three heads, two tails
   two heads, three tails
   one head, four tails

c) four heads $\rightarrow$ one microstate $\rightarrow$ $S \sim \ln(1) = 0$
   three heads $\rightarrow$ four microstates $\rightarrow$ $S \sim \ln(4) \approx 1.39$
   two heads $\rightarrow$ six microstates $\rightarrow$ $S \sim \ln(6) \approx 1.79$
   one head $\rightarrow$ four microstates $\rightarrow$ $S \sim \ln(4) \approx 1.39$
   zero heads $\rightarrow$ one microstate $\rightarrow$ $S \sim \ln(1) = 0$

d) There are more ways to get 2 heads, so I expect to see that.
e) The system will most likely go from 4 heads to 2 heads and entropy will increase from 0 to 1.79, in accordance with the second law.
f) If the system is in a low entropy microstate, we know information about individual particles; for example, we know the state of all four quarters if we measure the "four heads" macrostate. However, in a high entropy macrostate, we don't know about individual particles; for example, if we see that two quarters are heads up, we don't know which two are. Thus, from our point of view, the low entropy state is more "ordered" than the high entropy state.
2a) From the first law, we have $\Delta u = Q - W$. During a cycle, the engine returns to its initial conditions, so $\Delta u = 0$. So $Q = W$. But $Q$ is the total heat that enters the system. So $Q = Q_h + Q_c$. So $W = Q_h + Q_c = 100000 J + 8000 J = 20000 J$. 
$Q_c$ leaves the system, so it is negative.

b) $e = \frac{W}{Q_h} = \frac{20000}{100000} = 0.2$

c) $e_{max} = 1 - \frac{T_c}{T_h} = 1 - \frac{300}{5000} = 0.94$

d) Power = Work / Time = $\frac{20000 J}{0.015 s} = 200,000 W$

3a) The heat absorbed by the ice is $Q = mL = 2 kg \times 3.34 \times 10^5 J/kg$
The temperature is $0^\circ C = 273 K$.
So $\Delta S = \frac{Q}{T} \approx 2446.89 J/K$

b) We must use $\Delta S = mC ln \left( \frac{T_2}{T_1} \right) = 2 kg \times 1.168 kJ/kgK \cdot ln \left( \frac{273.5}{273} \right) \approx 1408.01 J/K$

4. An isothermal process is reversible because the system is in thermal equilibrium the whole time.

5. In an adiabatic process, $Q = 0$. So we can break it into tiny constant temperature steps, but at each one, $\Delta S = 0$, so the total entropy change during the process is zero.

6. $e = \frac{output}{cost}$. For a refrigerator, the cost is work and the output is the heat removed from the cold reservoir, so $e = \frac{Q_{c1}}{W} = \frac{Q_c}{W} = \frac{1}{\frac{Q_{c1}}{W} - 1}$

For a Carnot engine, $e = 1 - \frac{T_0}{T_1}$ or $e_{max} = 1 - \frac{T_c}{T_h}$
So we essentially replaced $\frac{Q_{c1}}{W}$ with $\frac{T_c}{T_h}$.
So for a fridge, $e_{max} = \frac{T_h}{T_h - T_c}$ or $e_{max} = \frac{T_c}{T_h - T_c}$.