1 Vector Basics

- Notation: \( \vec{A} = A_x \hat{x} + A_y \hat{y} + A_z \hat{z} = A_x \hat{i} + A_y \hat{j} + A_z \hat{k} = (A_x, A_y, A_z) \)
- Magnitude/Length: \( |\vec{A}| = A = \sqrt{A_x^2 + A_y^2 + A_z^2} \)
- Direction (for 2-D vector, measured counterclockwise from positive x-axis): \( \theta = \arctan \frac{A_y}{A_x} \)
- Addition: \( \vec{A} + \vec{B} = (A_x + B_x) \hat{x} + (A_y + B_y) \hat{y} + (A_z + B_z) \hat{z} \)
- Dot product (produces a scalar): \( \vec{A} \cdot \vec{B} = A_x B_x + A_y B_y + A_z B_z = AB \cos \theta \)
- Cross product (produces a vector): \( |\vec{A} \times \vec{B}| = AB \sin \theta \) with direction given by right hand rule

2 Translational and Rotational Dynamics

- \( v_{avg} = \frac{\Delta x}{\Delta t} \)
- \( a_{avg} = \frac{\Delta v}{\Delta t} \)
- \( x = x_0 + v_0 t + \frac{1}{2} a t^2 \)
- \( v = v_0 + at \)
- \( v^2 = v_0^2 + 2a \Delta x \)
- \( F = m \ddot{a} \)
- \( W = \int \vec{F} \cdot d\vec{r} = F_{avg} \Delta r \cos \theta \) -> (you didn’t learn this rotational form)
- \( KE_{trans} = \frac{1}{2} mv^2 \)
- \( \vec{p} = mv \)
- \( J = \Delta \vec{p} = F_{avg} \Delta t \) -> (you didn’t learn this rotational form)

3 Links Between Translation and Rotation

- Position (arc length formula): \( s = r\theta \) (actually \( \vec{s} = \vec{r} \times \vec{\theta} \))
- Tangential velocity: \( v = r\omega \) (actually \( \vec{v} = \vec{r} \times \vec{\omega} \))
- Tangential acceleration: \( a_t = r\alpha \) (actually \( \vec{a}_t = \vec{r} \times \vec{\alpha} \))
- Radial acceleration: \( a_r = r\omega^2 \) (consequence of centripetal acceleration)
- Moment of inertia for point particles: \( I = \sum_{i=1}^{N} m_i r_i^2 \)
- Moment of inertia for other shapes: \( I_{hoop} = MR^2, I_{disk} = \frac{1}{2} MR^2, I_{solid \ sphere} = \frac{2}{5} MR^2, I_{spherical \ shell} = \frac{2}{3} MR^2, I_{bar \ about \ cm} = \frac{1}{3} ML^2, I_{bar \ about \ end} = \frac{1}{12} ML^2 \)
- Momentum: \( \vec{\ell} = \vec{r} \times \vec{p} \)
- Torque: \( \vec{\tau} = \vec{r} \times \vec{F} \)
4 Other Assorted Formulas (Special Forces, Collisions, Center of Mass)

- Normal force: symbolized by $F_N$, points perpendicular to a surface and prevents an object from falling through it
- Spring force: $F = -kx$
- Gravitational force: points downward with magnitude $F = mg$
- Maximum static friction force: $f_s = \mu_s F_N$
- Kinetic friction force: $f_k = \mu_k F_N$
- Centripetal force: $\sum F_{\text{towards the center}} = \frac{mv^2}{r} = m\omega^2 r$
- Kinetic energy is conserved in elastic collisions, kinetic energy is not conserved in inelastic collisions, and objects stick together in perfectly inelastic collisions.
- Conservation of momentum: if the net external force on a system is zero, then $p_i = p_f$ (consequence of impulse formula) (also, if $\sum \vec{r}_{\text{ext}} = 0$, $\ell$ is conserved)
- Center of mass: $\vec{r}_{cm} = \frac{1}{M} \sum_{i=1}^{N} m_i \vec{r}_i$ where $M$ is the total mass of the particles

5 Work and Energy

- Translational kinetic energy: $KE_{\text{trans}} = \frac{1}{2} mv^2$
- Rotational kinetic energy: $KE_{\text{rot}} = \frac{1}{2} I \omega^2$
- The work done by a conservative force is independent of the path. Conservative forces have associated potential energies.
- Gravitational potential energy: $U_g = mgh$ (gravity is conservative)
- Elastic/spring potential energy: $U_k = \frac{1}{2} kd^2$ (spring force is conservative)
- Internal energy is symbolized by $U_{\text{int}}$
- Total mechanical energy: $E = KE_{\text{trans}} + KE_{\text{rot}} + U_g + U_k + U_{\text{int}} = KE + U$
- Work: $W = \int \vec{F} \cdot d\vec{r} = F_{\text{avg}} \Delta r \cos \theta$
- Work and total mechanical energy: $\Delta E = W_{\text{done on system}} \Rightarrow KE_f + U_f = KE_i + U_i + W$
- To keep track of signs, remember that if the work takes away energy (like friction) $W < 0$. Otherwise, $W \geq 0$.
- Conservation of energy: If no work is done, $E_i = E_f$ (consequence of previous two statements)