Moment of Inertia and Torque solutions

1. We have three objects, so in all cases, we just have to add their moments together.

   a) For the rod, we have $I_R = \frac{1}{12}ML^2$
      For each ball, we have $I_B = m\left(\frac{d}{2}\right)^2$
      So $I_{tot} = \frac{1}{12}ML^2 + 2m\left(\frac{L}{2}\right)^2 = \frac{1}{12}ML^2 + \frac{1}{2}ML^2$
      For $M = 4\text{ kg}$, $L = 2\text{ m}$, and $m = 0.5\text{ kg}$, $I = \frac{7}{3} = 2.33 \text{ kg.m}^2$

   b) For the rod: $I_R = \frac{1}{3}ML^2$
      For the left ball: $I_{BL} = mL^2 = 0$
      For the right ball: $I_{BR} = mL^2$
      So $I_{tot} = \frac{1}{3}ML^2 + mL^2 = \frac{2}{3}L^2 \approx 7.33 \text{ kg.m}^2$

   c) $I_{tot} = 0$ because all the mass is right at the axis, so all the $I_a$'s in the definition are zero.

   d) We have to use the parallel axis theorem.
      So $I = I_{cm} + (M+2m)d^2$. But $I_{cm} = 0$ (from the last problem).
      So $I = (M+2m)d^2$
      So $I = \frac{5}{4} = 1.25 \text{ kg.m}^2$

2. a) Use energy. So $E_i = E_f \Rightarrow mgh = \frac{1}{2}mv^2 \Rightarrow v = \sqrt{2gh}$

   b) Use energy again. So $E_i = E_f \Rightarrow Ug = kE$. But this time, the cylinder is rotating, so $kE = kE_{\text{linear}} + kE_{\text{rot}}$.
      So we have $mgh = \frac{1}{2}mv^2 + \frac{1}{2}I\omega^2$. We know that $I = \frac{1}{2}mR^2$.
      We also know that $v = WR$ since it rolls without slipping.
      So $mgh = \frac{1}{2}mv^2 + \frac{1}{2}\left(\frac{1}{2}mR^2\right)\left(\frac{h}{R}\right)^2$
      $\Rightarrow mgh = \frac{1}{2}mv^2 + \frac{1}{2}mv^2 = \frac{3}{4}mv^2 \Rightarrow v = \sqrt{\frac{3}{2}gh}$
3. a) We know that $\tau = I\alpha$ and that for a solid sphere, $I = \frac{2}{5}MR^2$. So $\tau = \frac{2}{5}MR^2\alpha$. To find $\alpha$, we use $\Delta\theta = \omega t + \frac{1}{2}\alpha t^2$. We know that $\Delta\theta = 720^\circ = \frac{4\pi}{15}$ radians, $\omega = 0$, and $t = 4s$. So $4\pi = \frac{1}{2}\alpha \cdot 16$. So $\alpha = \frac{\pi}{5}$ rad/s$^2$. So $\tau = \frac{2}{5} (0.2)(0.04)^2 \left(\frac{\pi}{5}\right) = \frac{\pi}{15625} \approx 0.000201 \text{ N} \cdot \text{m}$

Note that N·m $\neq$ J when we talk about torque!

b) We know that $\vec{\tau} = \vec{F} \times \vec{r} \Rightarrow |\vec{\tau}| = |\vec{F}||\vec{r}| \sin \theta$. If the force is applied $\perp$ to the surface, it is $\perp$ to the position vector. So $\theta = 90^\circ \Rightarrow \sin \theta = 1$.

So $|\vec{r}| = |\vec{F}| / 1\text{t} = \frac{\pi}{15625} \approx 0.005027 \text{ N}$

4. a) Let's set up the problem like this:

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com of the seesaw
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So we have to solve for $x$. Since the rod is uniform, we know that its center of mass is right in the middle. The torque produced by the 400 N child is just $-400x$. The torque is negative since it wants to rotate the seesaw clockwise. The other child produces a torque of $350(l-x)$, and we need the distance between the COM of the seesaw and the fulcrum. This is just $(l-x) = \frac{l}{2} = \frac{l}{2} - x$. So it produces a torque of $30 \times 9.8 \times \left(\frac{l}{2} - x\right)$. So the total torque is $350(l-x) - 400x + 294\left(\frac{l}{2} - x\right)$. Newton's Law for torques says that $\Sigma \tau = \pm \alpha$. In equilibrium, $\alpha = 0$, so $\Sigma \tau = 0$, so $350l - 350x - 400x + 147l - 294x = 0$.

So $497l - 1044x = 0 \Rightarrow 1044x = 497l \Rightarrow x = \frac{497l}{1044}$

So $x = 1.19 \text{ m}$. So place the fulcrum 1.19 m from the fat kid.
b) the fulcrum has to be at the center of mass. So an easier way to answer the question is this:

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x_{cm} = \frac{400 \times 0 + 30 \times \frac{9}{2} + 350 \times \frac{11}{4}}{400 \frac{9}{8} + 30 + 350 \frac{9}{4.8}} = 1.19 \text{ m}
\] from the fat kid