Potential Energy Solutions

1. We know that Power = \( \frac{W}{t} \), so we need work and time. We know that \( W = F \cdot d \), so we need to find \( F \). We know that \( F_{push} - F_f = ma \). We know \( m \) and \( a \), so we need to find \( F_f \). We know \( F_f = \mu_k F_n \). We know \( \mu_k \), so we need to find \( F_n \). We know that \( F_n - F_g = 0 \) and that \( F_g = mg \). So now we can solve for \( W \): 
   \[ W = F \cdot d = (F_f + ma) \cdot d = (\mu_k mg + ma) \cdot d = dm(a + \mu_k g) \] 
   To find time, we know that \( d = \frac{1}{2}at^2 \). So \( t = \sqrt{\frac{2d}{a}} \).
   So Power = \( \frac{d \cdot m \cdot (a + \mu_k g)}{\sqrt{\frac{2d}{a}}} \).
   For \( d = 2m \), \( m = 10 kg \), \( a = 2 m/s^2 \), \( \mu_k = 0.25 \), we have power \( \approx 63 \) watts, about the same as a light bulb.

2. 
   a) \( U_g = mgh \). For \( m = 500 kg \) and \( h = 20 m \), \( U_g = 98000 J \)
   b) \( W = F \cdot d \). The force is the frictional force and the distance is the height. So, for \( F = 300 N \) and \( d = 20 m \), \( W = -6000 J \)
   c) The total mechanical energy at the top is \( U_g \) and the total mechanical energy at the bottom is \( \frac{1}{2}mv^2 \). Without the brakes, total energy is conserved. So \( \frac{1}{2}mv_{brakes}^2 = U_g \Rightarrow v_{brakes} = \sqrt{\frac{2U_g}{m}} \approx 19.799 m/s \)
   When the brakes are applied, total energy is not conserved. In fact, we have \( E_{top} - E_{bottom} = \text{Work}_{friction} \).
   So \( U_g - \frac{1}{2}mv_{brakes}^2 = 6000 \Rightarrow \frac{1}{2}mv^2 = U_g - 6000 \Rightarrow v = \sqrt{\frac{2(U_g - 6000)}{m}} \)
   d) \( v_{brakes} \approx 19.1833 m/s \). So \( v_{brakes} \approx 0.9689 \cdot v_{brakes} \) 
   e) This ratio will change with \( m \) and \( h \), since \( \frac{v_{brakes}}{v_{brakes}} = \sqrt{1 - \frac{F}{mgh}} \) (prove this).
3.

a) \( U_{\text{top}} = mgh_n \)

b) If the ball compresses the spring \( d \), it also goes down a distance \( d \) below zero. So \( U_{\text{bottom}} = -mgd + \frac{1}{2}kd^2 \).

c) The ball has traveled a distance \( d \) with a frictional force \( f \). So it has lost \( -W_f = -(fx-d) = fd \).

d) The change in total mechanical energy is equal to the work lost to friction. So \( E_{\text{top}} - E_{\text{bottom}} = fd \). Kinetic energy is zero at the top and bottom, so we have \( U_{\text{top}} - U_{\text{bottom}} = fd \).

So \( mgh_n - (-mgd + \frac{1}{2}kd^2) = fd \)

So \( \frac{1}{2}kd^2 - (mg-f)d - mgh_n = 0 \).

Using the quadratic formula,

\[
d = \frac{-(mg-f) \pm \sqrt{(mg-f)^2 - 4\left(\frac{1}{2}k\right)(-mgh_n)}}{2\left(\frac{1}{2}k\right)}
\]

Since \( f \geq mg \), \( mg-f \geq 0 \). So we want the positive part of the symbol. So

\[
d = \frac{mg-f + \sqrt{(mg-f)^2 + 2kmgh_n}}{k}
\]

e) When the ball bounces back up, it will have lost \( 2fd \) to friction.

f) So \( mgh_n - 2fd = mgh_{n+1} \) \( \Rightarrow h_{n+1} = h_n - \frac{2fd}{mg} \)

So \( h_{n+1} = h_n - 2F \left[ \frac{mg-f + \sqrt{(mg-f)^2 + 2kmgh_n}}{mg} \right] \)

g) As \( h_n \) becomes smaller and smaller, \( \sqrt{(mg-f)^2 + 2kmgh_n} \) becomes approximately \( \sqrt{(mg-f)^2} = mg-f \). So

\[
h_{n+1} \approx h_n - 2f (mg-f + mg-f) = h_n - \frac{4f}{mg} (mg-f) = h_n - \frac{4f}{mg} + \frac{f}{mg}
\]

Since \( f \leq mg \), \( \frac{f}{mg} \leq 1 \), so \( h_{n+1} \) will keep getting less by \( \frac{4f}{mg} \) each bounce. Eventually, \( h_{n+1} \) will be zero and the ball will stop.