Newton’s Laws With Friction

1. A rectangular block of mass $M$ is sitting on an inclined plane of angle $\theta$, as shown below. The block is connected by a string to a sphere of mass $m$ such that $M < m$. The coefficient of static friction between the block and the plane is $\mu_s$, and the coefficient of kinetic friction between the block and the plane is $\mu_k$. The acceleration due to gravity is $g$, as always.

(a) Suppose that the block is just on the verge of moving. Find $k$, the ratio of the masses. ($k = m/M$)
(b) Suppose that $m = kM$ for the $k$ you just found, but that the block is now moving. Find its acceleration.

2. A block of mass $m$ sits on another block of mass $M$, as shown below. A horizontal force of magnitude $F$ acts on the lower block. The floor is frictionless, but the coefficient of static friction between the two blocks is $\mu_s$. What is the maximum value of $F$ that can be applied before the top block slides off?

3. A horizontal force of magnitude $F$ is applied to a pulley of mass $m$ which has a massless rope going over it, as shown below. The rope is connected to a wall on one end and to a block of mass $M$ on the other. The coefficient of kinetic friction between the ground and the block is $\mu_k$. Assume that there exists a constant vertical force on the pulley which supports its weight (i.e., the pulley moves only horizontally) and that the surface of the pulley is frictionless (i.e., the rope slides along it without causing it to rotate). Call the resulting acceleration of the block $a_B$ and the resulting acceleration of the pulley $a_P$.

(a) Use kinematics to find an expression relating $a_P$ to $a_B$. (Hint: If the pulley moves $\Delta x$ to the right during time $t$, how far does the block move during the same time?)
(b) Find $a_P$ and $a_B$ in terms of the other constants.
(c) Place a restriction on $F$ to ensure that the situation makes physical sense.