Superposition of Waves

1 Topics Covered

- **Superposition Principle**: If two waves \( W_1(x, t) \) and \( W_2(x, t) \) are moving in the same medium, the resulting waveform is the sum of the waves, i.e., \( W_{tot}(x, t) = W_1(x, t) + W_2(x, t) \). This sounds simple, but wave addition follows some strange arithmetic.

- **Interference**: If we have two waves of amplitude \( A_1 \) and \( A_2 \), we can use trig formulas to show that the amplitude of the resulting waveform can be anywhere between \( |A_1 - A_2| \) and \( A_1 + A_2 \). When the amplitude is \( |A_1 - A_2| \), we call this *destructive interference*, and when the amplitude is \( A_1 + A_2 \), we call this *constructive interference*.

- **Interference from Position**: Suppose there are two speakers emitting the same sound, i.e., they have the same \( k \) and \( \omega \) but that you are different distances from each. Let’s say that the difference is given by \( \Delta x = x_2 - x_1 \). Then:
  1. You will experience destructive interference when \( k\Delta x = n\pi \), where \( n \) is any odd integer
  2. You will experience constructive interference when \( k\Delta x = n\pi \), where \( n \) is any even integer

- **Interference from Frequency**: Suppose two instruments equidistant from you are playing two different notes, i.e., there are two waves with the same \( k \) but different \( \omega \). Let’s say the difference is given by \( \Delta \omega = \omega_2 - \omega_1 \). Then:
  1. You will experience destructive interference when \( t\Delta \omega = n\pi \), where \( n \) is any odd integer
  2. You will experience constructive interference when \( t\Delta \omega = n\pi \), where \( n \) is any even integer. These constructive interferences are called beats.
  3. The frequency at which you hear the beats will be given by \( f_{\text{beat}} = \Delta \omega / 2\pi \).

- **Standing Waves**: If we have two waves with the same amplitude, wavenumber, and frequency that move in opposite directions through the same medium, we can create standing waves (provided that certain “boundary conditions” are met). We will consider standing sound waves in tubes of air.

- **Tube Open at One End**: Suppose there is a sound wave traveling through a tube of length \( L \) that is open at \( x = L \) and closed at \( x = 0 \). There will be a standing wave if and only if \( 2kL = n\pi \), where \( n \) is an odd integer greater than zero.

- **Tube Open at Both Ends**: Suppose there is a sound wave traveling through a tube of length \( L \) that is open at \( x = L \) and open at \( x = 0 \). There will be a standing wave if and only if \( 2kL = n\pi \), where \( n \) is an even integer greater than zero.

- **Harmonics**: The frequency that corresponds to the smallest value of \( n \) is called the fundamental frequency. The frequency that corresponds to the next smallest value of \( n \) is the first harmonic, then comes the second harmonic, etc.

- **Doppler Effect**: If either a source of sound or a listener is moving with respect to the medium that carries the sound, the listener will perceive a different frequency than what was emitted. Let \( v \) be the speed of sound, \( v_S \) be the speed of the source with respect to the medium, and \( v_L \) be the speed of the listener with respect to the medium. (The direction from the listener to the source is taken to be positive.) If sound is emitted at a frequency \( f_0 \), the sound heard by the listener will have frequency \( f = f_0 \left( \frac{v + v_L}{v + v_S} \right) \).
2 Problems

1. A wave of amplitude 4 and a wave of amplitude 3 are traveling in the same medium. What is the maximum and minimum possible amplitude of the resulting wave?

2. Two violinists are playing together, but they are slightly out of tune; one produces a frequency of 883 Hz and the other produces a frequency of 879 Hz. What is the beat frequency?

3. Suppose you have a tuning fork that makes a sound at a certain frequency, and you want to tune a piano to the same frequency. How can you use beats to accomplish this?

4. You are standing still on a sidewalk as a police car goes by in hot pursuit. The speed of sound is 344 m/s, the frequency of the police siren is \( f_0 = 10 \text{ kHz} \), and the speed of the police car is 70 mph. What is the change in frequency that you hear as the car goes by? Does the sound get higher or lower?

5. Suppose you walk along the line that connects two speakers that are 15 m apart. The speed of sound is the usual 340 m/s, and the speakers are playing a constant bass note at 100 Hz.

   (a) Find the wavenumber and wavelength of the sound wave.

   (b) If you start walking from one speaker, how far must you travel before you experience destructive interference?

   (c) If you walk all the way from one speaker to the other, how many times will you experience constructive interference?

6. Is a guitar string like a tube that is open at both ends or a tube that is open at one end?

7. If a cello string is 0.5 m long, what is the wavelength of its third harmonic?